# GPS-UTM Module 3:

## *Do We Have A Regulation Field?*

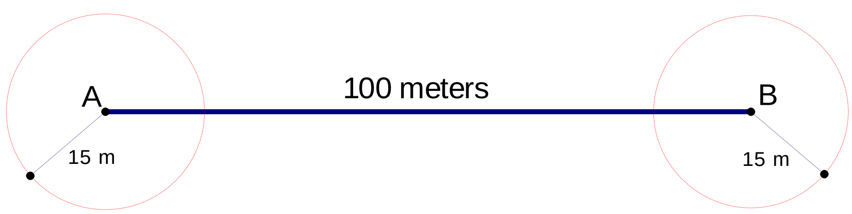
**Topics Covered:** Measurement error, determining right angles, parallelograms

**Required Background Material:** GPS-UTM Module 2, familiarity with right triangles and polygons

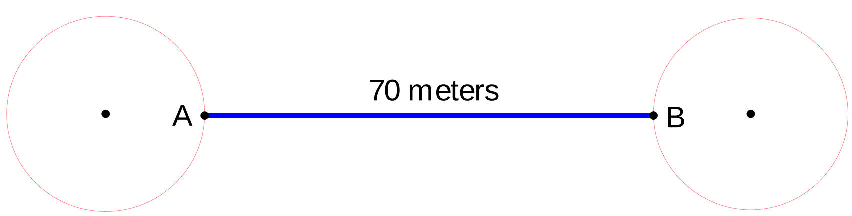
## Introduction

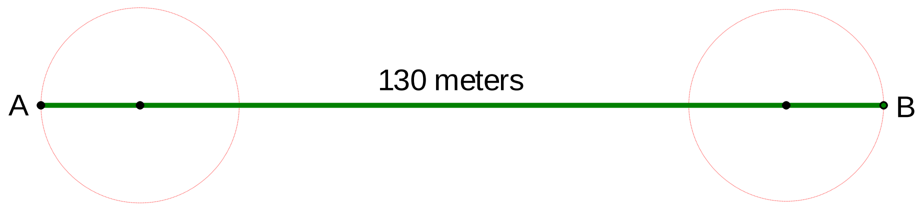
Today's GPS receivers are extremely accurate compared to those used in the 20th Century, but they can still give measurements that are up to 15 meters away from the exact location of the receiver. GPS accuracy also depends on the location of satellites in the sky, and on interference from buildings and heavy clouds. In this module we will discuss how the accuracy of located points can affect distance calculations.

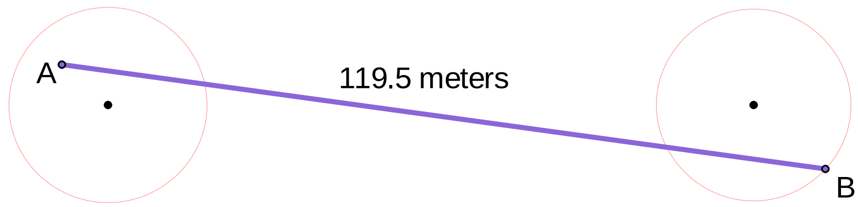
In the diagram below, look at points A and B which we have supposedly calculated to be 100 meters apart from each other. However, the actual position of the points could be anywhere within circles of radius 15 meters, centered at A and B.




Consider some other possibilities for the location of points A and B, as shown in the following three diagrams.





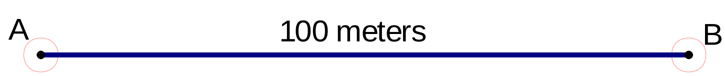

It should be pretty clear that the smallest possible distance between A and B would be $100 - 2(15) = 70\ meters$, and the largest possible distance between A and B would be $100 + 2(15) = 130\ meters$. If we let $d$ be the distance from A to B, we can then say:

$$70 \leq d \leq 130$$

In interval notation, we say that $d \in [70,130]$. $[70,130]$ is called a *confidence interval for d.*

**Problem 1**

The GPS unit you are using probably contains a Wide Area Augmentation System (WAAS). If the display on your device indicates that the accuracy is less than ten meters, then you are automatically in WAAS mode. If not, you may be able to set the receiver in WAAS mode. (Ask your teacher how to do this.) On a clear day, you might then be able to measure distances to within 3 meters as shown in the diagram below.




If $d$ is the distance from A to B, find the smallest and largest possible actual value if the calculated value is 100 meters.

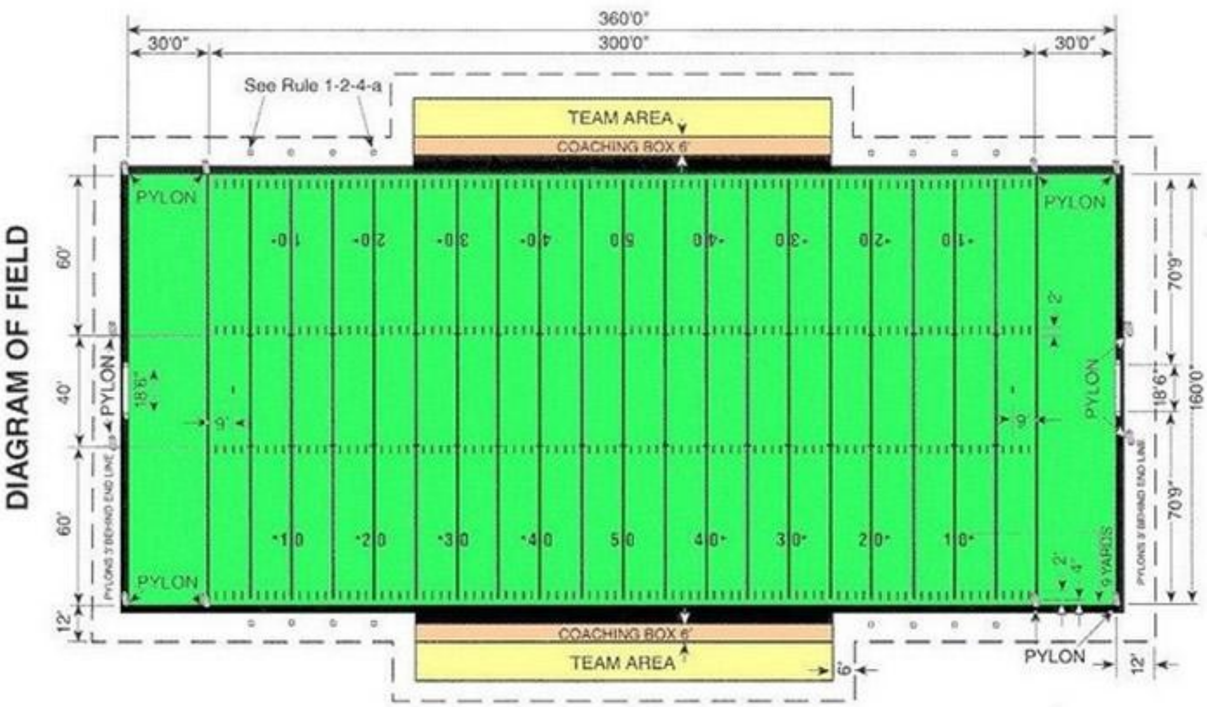
$$\_\_\_\_\_\_\ meters \leq d \leq \_\_\_\_\_\_\ meters$$

What is the confidence interval for *d*?

If the calculated value was 50 meters, what would the confidence interval be?

**Problem 2**

Look at the standard football field diagram below. Our goal in this module is to use GPS technology to find out if the football field at your high school is a regulation field.




**Regulation Football Field**

The diagram above shows that a regulation field is 300 feet (100 yards) from goal line to goal line. The width of the field is 160 feet. Since our GPS devices measure distance in meters, we need to convert the dimensions to meters. Complete the following chart, using the fact that 1 yard = 0.9144 meter.

| | Feet | Yards | Meters |
|---|---|---|---|
| **Length** | | | |
| **Width** | | | |

## Problem 3

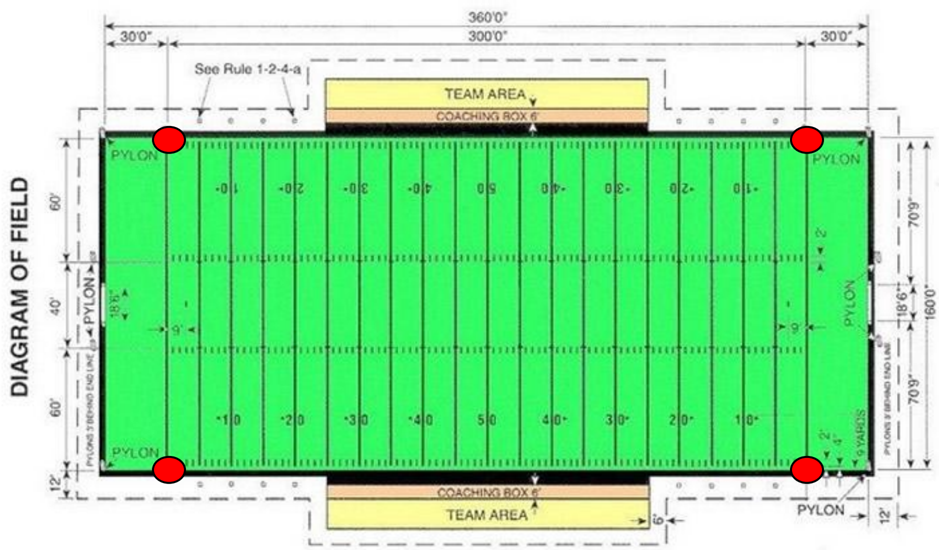
If the length and width of a football field are measured using a GPS device in WAAS mode, what are the confidence intervals in meters for the two measurements?

**Confidence Interval for the Length:**

**Confidence Interval for the Width:**

## Problem 4

Put your GPS device into WAAS mode, go to the football field at your high school, and record the UTM coordinates of the four corner points (where the pylons are on the goal lines). These points are indicated by red dots in the diagram below.

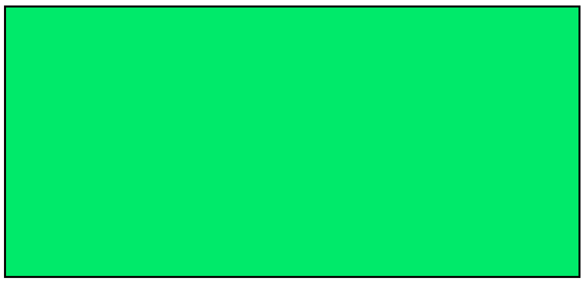



List your data in the chart below.

| Point | Northing | Easting |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |

**Problem 5**

On the diagram below, label the points by number. Then write the distances in the chart, and indicate whether each measured distance is within the appropriate confidence interval as found in Problem 3. Finally, convert the distances to yards and write the yardage distances along the appropriate sides in the diagram.

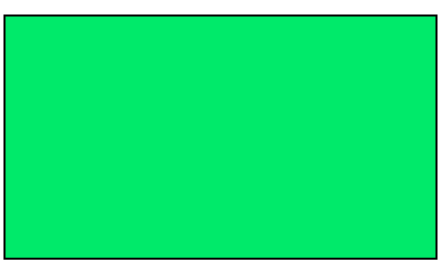


| Side | Length in Meters | Within the Confidence Interval? |
|---|---|---|
| d(Point 1, Point 2) | | |
| d(Point 2, Point 3) | | |
| d(Point 3, Point 4) | | |
| d(Point 4, Point 5) | | |

Are the dimensions of your high school football field within regulation?

**Problem 6**

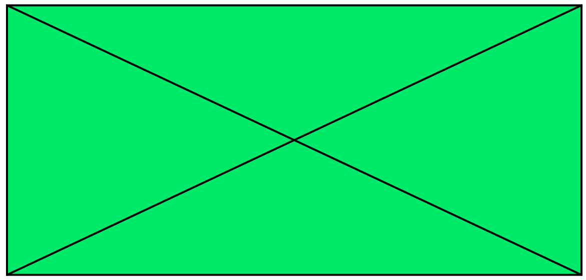
A regulation football field also has to be rectangular; it has to have $90^{\circ}$ angles at the corners. Without right angles in the corners, it could have opposite sides of equal measure yet not be a rectangle. Such a figure is called a parallelogram.



**Rectangle**

**Parallelogram**

We can check for right angles by using the Pythagorean Theorem and the diagonals as drawn below.



Use the lengths you found in Problem 5 and the Pythagorean Theorem to find the lengths of the two diagonals for the football field. Also find the lengths directly from the UTM coordinates in Problem 4. Lastly find the lengths that the diagonals should be, using the regulation measurements in Problem 2. Put all of those numbers into the chart below.

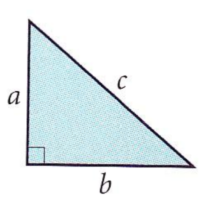
| **Diagonal** | **Length using Problem 5 and the Pythagorean Theorem** | **Length using the UTM Coordinates in Problem 4** | **Length in Meters for a Regulation Field** |
|---|---|---|---|
| **d(Point 1, Point 3)** | | | |
| **d(Point 2, Point 4)** | | | |

Does your football field appear to be within regulation?

If your field does not appear to have right angles on all of the corners, which angles are too big and which angles are too small?

## Problem 7 (Opt.)

You may have noticed that the numbers in the first two columns of Problem 6 are slightly different. That is because the numbers in the first column rely on 3 measured points, while each number in column two uses only 2 measured points. Go back and look at the Pythagorean Theorem again.




$$c^2 = a^2 + b^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

If $a$ and $b$ are measured with a GPS device in WAAS mode, their lengths could be off by as much as 6 meters. Using the Greek letter $\Delta$ (delta), we say that $\Delta a = 6$ meters and $\Delta b = 6$ meters. Using $c^2 = a^2 + b^2$, the branch of mathematics called *calculus* gives the following formula for $\Delta c$:

$$\Delta c = \frac{a \cdot \Delta a + b \cdot \Delta b}{\sqrt{a^2 + b^2}}$$

Using the values (in meters) for the length $a$ and width $b$ of the field, find the expected error ($\Delta c$) in computing the diagonal $c$.

$$\Delta c = \underline{\hspace{4cm}}$$

Your answer should be larger than 6 meters, the allowed GPS error for the calculations in column 2. The number is larger because of what is called *propagation of error.* It comes from using approximate numbers to approximate other numbers.