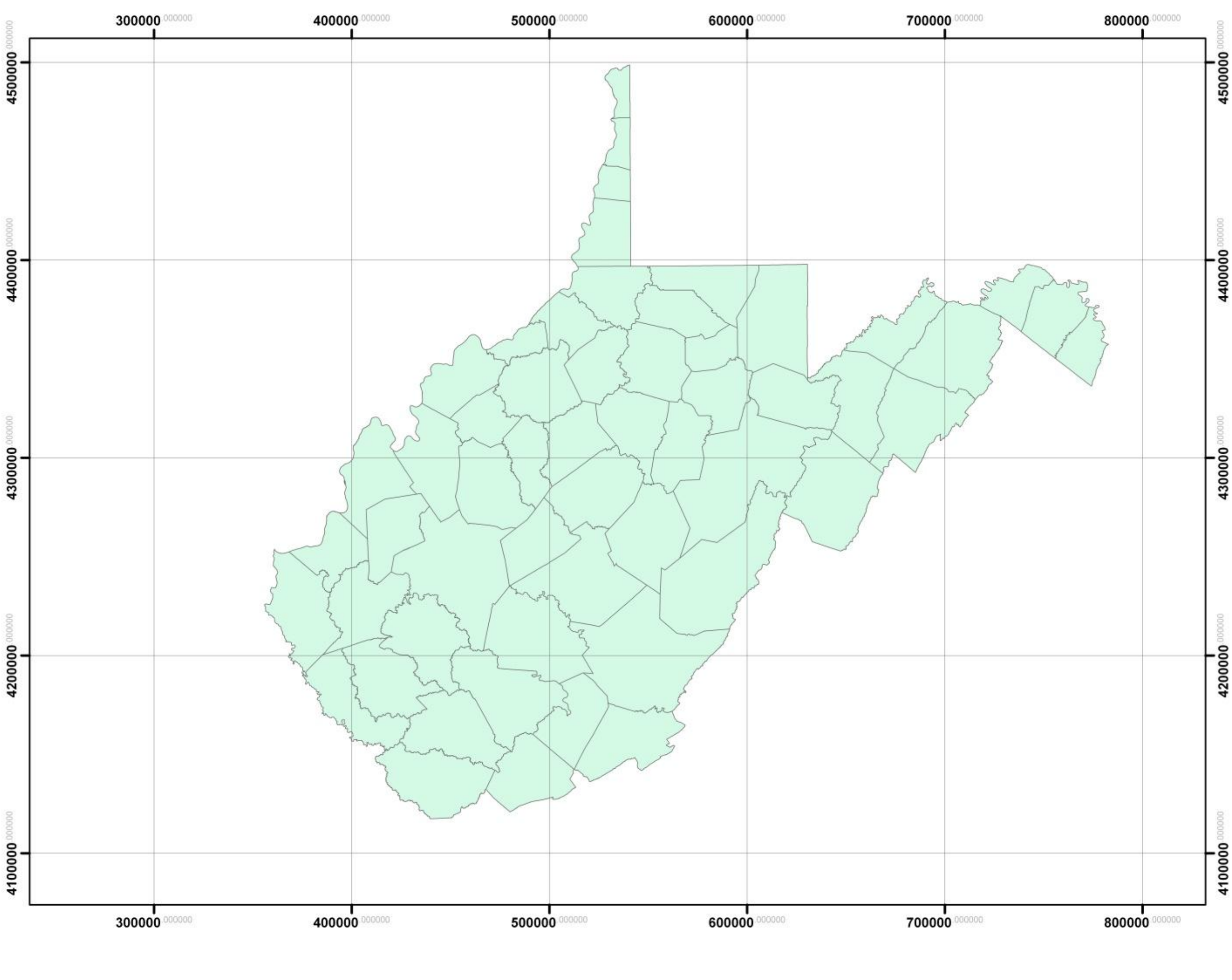
GPS-UTM Module 4:
What is a Googleplex?

Topics Covered: Scientific notation, comparison of large and small numbers

Required Background Material: GPS-UTM Module 2, familiarity with the laws of exponents and with rounding techniques

<u>Introduction</u>

There will be no GPS fieldwork in this unit. Instead we will discuss how to compare distances and we will take a close look at the numbers on a UTM grid. Here is the UTM map of West Virginia as shown in Module 1.



The many zeros around the edges make this map a bit confusing. Notice, first of all, that the small zeros in light gray represent digits to the right of the decimal point. In other words,

500000^{000000} = 500,000.000000 = five hundred thousand.

Problem 1

One of the numbers on the side of the grid is **4400000**^{000000}. Write this number in both in words and as a decimal number.

Grid Number	Decimal Form	Word Form
4400000^{000000}		

Scientific Notation

Generally, when numbers are written to the right of the decimal point, they indicate that the decimal part of the number is *significant.* That means that 500,000.000000 lies between 499,999.9999995 and 500,000.00000049999999⋯. It is rounded to the nearest .000001. (Recall that to round to the 6th digit to the right of the decimal point, you look at the 7th digit. Round up if the 7th digit is 5 or higher.)

The significance of the zeros to the right of the decimal point is not obvious when we use the word form. In fact, five hundred thousand is usually just written as 500,000. Clearly the decimal form tells us more about the accuracy of the number than the word form does.

But how do we know which zeros in the number 500,000 are significant? To be more specific, what if we wanted to say that the number is between 499,950 and 500,049? One way to do that would be to write the decimal number as 500,$\underline{0}$00. The underlined zero is tells us that the number has been rounded to the nearest hundred.

An easier way, and one that uses much less effort when there are lots of zeros, is to use ***scientific notation***. In scientific notation we write all numbers with just one digit to the left of the decimal point. All of the other significant digits would be on the right, and the size of the number will be shown as a power of ten. Thus if 500,000 is accurate to the nearest hundred thousand, we can write it as 5×10^5 (since $500{,}000 = 5 \times 100{,}000$). And if 500,000 is accurate to the nearest hundred, we can write it as 5.000×10^5.

Here is another example: $328 = 3.28 \times 100 = 3.28 \times 10^2$

Problem 2

Write the following numbers in scientific notation.

Number	Scientific Notation
12	
4400000^{0000}	
32,0000	

Problem 3

Consider the three different ways to writing number forms, as listed in the table below. Write *good, neutral,* or *bad* in the six empty boxes, to evaluate whether each type of number is easy to write and whether, in general, it shows the correct accuracy.

Type of Number	Shows Correct Accuracy	Is Easy to Write
Word form		
Decimal form		
Scientific notation		

Problem 4

Scientific notation is particularly handy when dealing with very large or very small numbers. Because we are focusing on GPS distances in this lesson, we will only work with large numbers.

Here are some large numbers which have specific names:

Decimal Number	Scientific Notation	Name
1000	10^3	thousand
1,000,000	10^6	million
1,000,000,000	10^9	billion
1,000,000,000,000	10^{12}	trillion

Eventually, for very large numbers, scientist and mathematicians run out of names and have to rely on scientific notation. The largest named number is a *googleplex.* Look it up on the worldwide web, using the *Google* search engine, and write the number in scientific notation.

1 *googleplex* = ___________________

Why do you think the name *Google* is used for a web search engine?

Can you type a googleplex in decimal form? Why, or why not? [Hint: Assume that you can type for 100 years at 1000 pages per day and that each page has 100 characters per line and 50 lines to a page.]

If all of the people in the world lived to be 100, and all of them typed all day, how many googleplexes could they type in decimal form?

Comparing Numbers

Suppose we wanted to compare the number *one billion* to the number *one million.* The best way to do this is to form a ratio, with the largest number on the top.

$$\frac{one\ billion}{one\ million} = \frac{10^9}{10^6} = 10^{9-6} = 10^3 = 1000$$

Thus one billion is a thousand time larger than one million.

Here is another example:

$$\frac{150\ billion}{3\ million} = \frac{150\times10^9}{3\times10^6} = \frac{150}{3}\times\frac{10^9}{10^6} = 50\times10^3 = 5\times10^1\times10^3 = 5\times10^4 = 50{,}000$$

Thus 150 billion is fifty thousand times as big as 3 million.

Problem 5

The distance across the United States is approximately 3000 miles. Convert this distance to meters and fill out the following chart in scientific notation. You may use the UTM maps on pp. 5 & 6 of Module 1 in order to estimate the width of West Virginia and Greenbrier County.

Geographic Object	United States	West Virginia	Greenbrier County
Width in Meters			

Compare these three widths, as shown in the following table.

Width of US/Width of WV	
Width of US/Width of Greenbrier County	
Width of WV/Width of Greenbrier County	

Problem 6 (Opt.)

Measure the length of your classroom in feet. Using your data from Problem 5, make a chalk line on the floor that represents the width of the state of West Virginia so that it is in correct proportion if the length of the classroom represents the width of the United States. Next, make a second chalk line that represents the width of Greenbrier County, and then fill out the following chart.

	Length in Feet
Classroom (Representing the US)	
Line Representing West Virginia	
Line Representing Greenbrier County	